Malaysian Journal of Science **34** (2) : 208-213 (2015)

ON SUBSPACE-DISK TRANSITIVITY OF BILATERAL WEIGHTED SHIFTS

Nareen Bamerni [1] and Adem Kilicman[2*]

[1,2] Department of Mathematics, University Putra Malaysia, 43400 UPM, Serdang, Selangor, Malaysia
[*] Corresponding Author: akilicman@yahoo.com

ABSTRACT In this paper, we give some properties of subspace-disk transitive operators and use them to characterize subspace-disk transitive bilateral weighted shifts in terms of their weight sequences. As a consequence, we show that a bilateral weighted shift is subspace-disk transitive if and only if it satisfies the subspace-diskcyclic criterion. Then, we give a much simpler condition for some other cases.

(**Keywords**: subspace-diskcyclic operators, subspace-disk transitive operators, weighted shifts)

INTRODUCTION

A bounded linear operator T on a separable Hilbert space H is called hypercyclic if there exists a vector $x \in H$ such that the orbit $Orb(T, x) = \{T^n x : n \geq 0\}$ is dense in H. Such a vector x is called hypercyclic for T.

In 1969, Rolewicz provided the first example of a hypercyclic operator on a Banach space [1]. In particular, he showed that if B is the backward shift on $l^p(N)$ then λB is hypercyclic if and only if λ has a modulus greater than one. Based on this example the supercyclicity notion was introduced by Hilden and Wallen in 1974 [2]. An operator T is called supercyclic if there exists a vector $x \in H$ such that the cone generated by $Orb(T, x)$ is dense in H, such a vector x is called a supercyclic vector for T. For more information about hypercyclicity and supercyclicity the reader may refer to [3], [4].

Also based on the Rolewicz example, the diskcyclicity concept was introduced by Zeana in 2002 [5]. An operator T is called diskcyclic if there is a vector $x \in H$ such that the disk orbit $DOrb(T, x)$ (where D is closed unit disk) is dense in$H$. Such a vector x is called diskcyclic for$T$. For more information about diskcyclicity, see [6], [7].

In 2011, Madore and Martınez-Avendano [8] studied the density of the orbit in a nontrivial subspace instead of the whole space and called that phenomenon the subspace-hypercyclicity. For the series of references on subspaces-hypercyclic operators see [8], [9], [10], [11].

Similarly, Xian-Feng et al. [12] studied the subspace-supercyclicity in a Hilbert space H that is the scaled orbit of an operator T is dense in a subspace of H. Also, in 2014, Bamerni and Kılıcman [13] introduced the subspace-diskcyclicity concept in a Hilbert space H that is the disk orbit of an operator T is dense in a subspace of H.

In 1995, Salas [14] characterized weighted shift operators that are hypercyclic or supercyclic. Moreover, Feldman [15] found some simpler conditions for the invertible weighted shift case. Also he showed that the same conditions hold true for some weaker cases.

In 2015, Bamerni et al. [6] posed necessary and sufficient conditions for bilateral weighted shift operators to be diskcyclic which were originally obtained by Zeana [5], and they showed that unilateral weighted shifts are diskcyclic if and only if they are hypercyclic.

In 2015, Bamerni and Kılıcman [16] characterized all weighted shifts operators that are subspace-transitive for some subspaces.

It is unknown when the bilateral weighted shifts operators could be subspace-diskcyclic. In section three of this paper, we give some equivalent assertions to subspace-disk transitivity. We use that results to obtain necessary and sufficient conditions for bilateral weighted shift operators on $l^2(Z)$ to be subspace-disk transitive. More precisely, we show that a bilateral weighted shift satisfies subspace-diskcyclic criterion if and only if it satisfies some certain conditions.

Moreover, we give some simpler conditions characterize the invertible bilateral weighted shifts that are subspace-disk transitive. Finally, through Theorem 3.6, we show that a bilateral weighted shift with a positive weight sequence $\{w_n\}_{n \in Z}$ $w_n \geq b > 0$ for all $n < 0$ is subspace-disk transitive if and only if the same simpler conditions hold true.

PRELIMINARIES

In this section, we will call some definitions and hypotheses that we need during the next section. We denote by H a complex Hilbert space and by B(H) the set of all bounded linear operators onH. All subspaces of H are assumed to be topologically closed.

Definition 2.1. [13] Let T be a bounded linear operator on a Hilbert space H, and let M be a nontrivial closed subspace of H. Then T is called subspace-disk transitive (M-disk transitive) for M if for any two non-empty relatively open sets U and V in M, there exist $n \in N$ and $\alpha \in U^c$ (where U is open unit disk) such that $T^{-n}(\alpha U) \cap V$ contains a non-empty relatively open set G of M.

Theorem 2.2. [13] Every subspace-disk transitive operator is subspace-diskcyclic for the same subspace.

Theorem 2.3 (Subspace-diskcyclic criterion). [13] Let $T \in B(H)$ and M be a subspace of H. Suppose that there is an increasing sequence of positive integers $\{n_k\}$, and there are two dense sets $D_1, D_2 \in M$ such that the following properties hold.

1) For any $y \in D_2$, there is a sequence $\{x_k\}$ in M such that $\|x_k\| \to 0$ and $T^{n_k} x_k \to y$;
2) $\|T^{n_k} x_k\|\|x_k\| \to 0$ for all $x \in D_1$;
3) $T^{n_k} M \subseteq M$ for all $k \in N$.

Then T is an M-disk transitive operator.

Theorem 2.4. [16] Let T be an invertible bilateral weighted shift, $\{n_k\}$ be a sequence of positive integers such that $n_k \to \infty$ as $k \to \infty$ and M be a closed subspace such that $T^{n_k} M \subseteq M$. If there exists an $i \in N$ such that $T^{n_k} e_{m_i} \to 0$ as $n_k \to \infty$, then $T^{n_k} e_{m_r} \to 0$ for all $r \in N$.

MAIN RESULTS

First we will give some characterization of subspace-disk transitive operators.

Proposition 3.1. Let $T \in B(H)$ and M be a non-trivial closed subspace of H. Then the following assertions are equivalent:

1) T is M-disk transitive,
2) For any two relatively open sets U and V in M, there exists $\alpha \in U^c$, $n \in N$ such that $T^{-n}(\alpha U) \cap V$ is nonempty and $T^n M \subseteq M$.
3) For any two relatively open sets U and V inM, there exists $\alpha \in U^c$, $n \in N$ such that $T^{-n}(\alpha U) \cap V$ is nonempty and open in M.

Proof: (1) ⇒ (2): Let U and V be two nonempty open subsets of M, and let α, n and G be from Definition 2.1, i.e. $G \subset T^{-n}(\alpha U) \cap V$ is nonempty and open in M. Then, $T^{-n}(\alpha U) \cap V$ is nonempty. Since $G \subset T^{-n}(\alpha U)$ it follows that $\frac{1}{\alpha} T^n G \subset U \subset M$. Let $x \in M$ and take $x_0 \in G$. There exists r > 0 such that $(x_0 + rx) \in G$. Then, we get $\frac{1}{\alpha} T^n x_0 + \frac{1}{\alpha} T^n rx = \frac{1}{\alpha} T^n (x_0 + rx) \in \frac{1}{\alpha} T^n G \subset M$.Since $x_0 \in G$ then $\frac{1}{\alpha} T^n x_0 \in \frac{1}{\alpha} T^n G \subset M$, it follows that $\frac{r}{\alpha} Tx \in M$ and so $Tx \in M$.

(2) ⇒ (3): Since $T^n|_M \in B(M)$, then $T^{-n}(\alpha U) \cap M$ is open in M for any U open subset of M. Since $V \subset M$ is open, it follows that $T^{-n}(\alpha U) \cap V$ is a nonempty open set in M.

3) ⇒ (1) is obvious.

Proposition 3.2. Let $T \in B(H)$. The following statements are equivalent:

1) T is M-disk transitive;
2) For all $x, y \in M$, there exist sequences $\{x_k\} \in M$, $\{n_k\} \in N$ and $\{\alpha_k\} \in D\setminus\{0\}$ such that for all $k \geq 1$, $T^{n_k} M \subseteq M$ and as $k \to \infty$, $\{x_k\} \to x$ and $T^{n_k} \alpha_k x_k \to y$;
3) For each $x, y \in M$ and each neighborhood W for zero in M, there exist $z \in M$, $n \in N$ and $\alpha \in D\setminus\{0\}$; such that $x - z \in W$, $T^n \alpha z - y \in W$ and $T^n M \subseteq M$.

Proof: (1) ⇒ (2): Let $x, y \in M$. For all $k \geq 1$, suppose that $B_k = B(x, 1/k)$ and $B'_k = B(y, 1/k)$ are two open balls inH, then $A_k = B_k \cap M$ and $A'_k = B'_k \cap M$ are relatively open subsets of M . By Proposition 3.1, there exist sequence $\{n_k\} \in N$, $\{x_k\} \in M$ and $\{\lambda_k\} \in U^c$ such that for all $k \geq 1$,

$$x_k \in T^{-n_k}(\lambda_k A'_k) \cap A_k \text{ and } T^{n_k} M \subseteq M.$$

It follows that

$$x_k \in A_k \text{ and } T^{n_k}\left(\frac{1}{\lambda_k} x_k\right) \in A'_k$$

Then, by taking$\alpha_k = \frac{1}{\lambda_k}$ and $k \to \infty$ the desired result follows.

(2) ⇒ (3): Follows immediately from part (2) by taking $z = x_k$ and $n_k = n$ for a large enough $k \in N$.

(3) ⇒ (1): Let U and V be two nonempty open subset of M. Let W be a neighborhood for zero, pick $x \in U$ and $\in V$, so there exist $z \in M$, $n \in N$ and $\alpha \in D\setminus\{0\}$; such that $x - z \in W$, $T^n(\alpha z) - y \in W$ and $T^n M \subseteq M$. It follows $U \cap T^{-n}(\frac{1}{\alpha} V) \neq \emptyset$ which proves (1), by Proposition 3.1.

The next two Theorems give necessary and sufficient conditions for a bilateral weighted shift operator on the Hilbert space $l^2(\mathbb{Z})$ to be M-disk transitive. We will suppose that

$$B = \{e_{m_r} : r \in N\}$$

is a Schauder basis for M, where $m_r \in Z$ and

$$F = \{m_r : r \in N, e_{m_r} \in B\}.$$

Let T be the bilateral forward weighted shift operator with a weigh sequence $\{w_n\}$, then $T(e_{m_r}) = w_{m_r} e_{m_r+1} + 1$ for all $r \in N$. The backward shift B is the right inverse to T which is defined as follow. $B(e_{m_r}) = \frac{1}{w_{m_r-1}} e_{m_r-1}$. Observe that $TB(e_{m_r}) = e_{m_r}$ for all $r \in N$. If T is invertible then $T^{-1} = B$. Note that for all $r \in N$ and $k \geq 0$, we have

$$T^k(e_{m_r}) = \left(\prod_{j=m_r}^{m_r+k-1} w_j\right) e_{m_r+k}$$

and

$$B^k(e_{m_r}) = \left(\prod_{j=m_r-1}^{m_r-k} \frac{1}{w_j}\right) e_{m_r-k}$$

Theorem 3.3. Let T be a bilateral forward weighted shift in the Hilbert space $l^2(Z)$ with a positive weight sequence $\{w_n\}_{n\in Z}$ and M be a closed subspace of H, then the following properties are equivalent:

1) T is M-disk transitive;
2) For all $q \in N$
   a. $\limsup_{n\to\infty} \min\left\{\prod_{k=1}^{n} w_{m_j-k} : m_j \in F \text{ and } |m_j| \leq q\right\} = \infty$;
   b. $\liminf_{n\to\infty} \max\left\{\frac{\prod_{k=0}^{n-1} w_{m_i+k}}{\prod_{k=1}^{n} w_{m_j-k}} : m_i, m_j \in F \text{ and } |m_i|, |m_j| \leq q\right\} = 0$;
   c. $T^n M \subseteq M$.
3) T satisfies M-diskcyclic criterion.

Proof: (1) ⇒ (2): Assume that T is M-disk transitive. Let $y = z = \sum_{\substack{|m_j|\leq q \\ m_j\in F}} e_{m_j} \in M$, then by Proposition 3.2, there exist an $x \in M$, an $n \geq q$, $\alpha \in D\setminus\{0\}$ and $0 < \epsilon < 1$ such that

$$T^n M \subseteq M, \tag{1}$$

$$\|x - z\| = \left\|x - \sum_{\substack{|m_j|\leq q \\ m_j\in F}} e_{m_j}\right\| < \epsilon \tag{2}$$

and

$$\|\alpha T^n x - y\| = \left\|\alpha T^n x - \sum_{\substack{|m_j|\leq q \\ m_j\in F}} e_{m_j}\right\| < \epsilon. \tag{3}$$

Inequality (2) implies that $|x_{m_j}| > 1 - \epsilon$ if $|m_j| \leq q$ and $|x_{m_j}| < \epsilon$ otherwise. Since $n > 2q$ inequality (3) implies that for $|m_j| \leq q$

$$|\alpha|\left|x_{m_j}\right|\left\|T^n e_{m_j}\right\| = |\alpha|\left|x_{m_j}\right|\left(\prod_{k=0}^{n-1} w_{k+m_j}\right) < \epsilon.$$

It follows that

$$\left(\prod_{k=0}^{n-1} w_{k+m_j}\right) < \frac{\epsilon}{|\alpha|\left|x_{m_j}\right|} < \frac{\epsilon}{|\alpha|(1-\epsilon)} = \frac{\delta}{|\alpha|}. \tag{4}$$

Where $\delta = \frac{\epsilon}{1-\epsilon}$. Also inequality (3) implies that $\left\|\alpha x_{m_j-n}\left(T^n e_{m_j-n}\right) - e_{m_j}\right\| < \epsilon$ for $|m_j| \leq q$. Thus

$$|\alpha|\left|x_{m_j-n}\right|\left|\prod_{k=0}^{n-1} w_{m_j-n+k} - 1\right| = |\alpha|\left|x_{m_j-n}\right|\left|\prod_{k=1}^{n} w_{m_j-k} - 1\right| < \epsilon.$$

Therefore

$$|\alpha|\left|x_{m_j-n}\right|\left(\prod_{k=1}^{n} w_{m_j-k}\right) > 1 - \epsilon. \tag{5}$$

and hence

$$\left(\prod_{k=1}^{n} w_{m_j-k}\right) > \frac{1-\epsilon}{|\alpha|\left|x_{m_j-n}\right|} > \frac{1-\epsilon}{|\alpha|\epsilon} = \frac{1}{|\alpha|\delta}. \tag{6}$$

Now, since $|\alpha| \leq 1$, then inequality (5) leads to

$$\left|x_{m_j-n}\right|\left(\prod_{k=1}^{n} w_{m_j-k}\right) > |\alpha|\left|x_{m_j-n}\right|\left(\prod_{k=1}^{n} w_{m_j-k}\right) > 1 - \epsilon.$$

Thus,

$$\left(\prod_{k=1}^{n} w_{m_j-k}\right) > \frac{1-\epsilon}{\left|x_{m_j-n}\right|} > \frac{1-\epsilon}{\epsilon} = \frac{1}{\delta}. \tag{7}$$

From equation (4) and equation (6), it follows that for all $|m_j| \leq q$,

$$\frac{\prod_{k=0}^{n-1} w_{k+m_j}}{\prod_{k=1}^{n} w_{m_j-k}} < \left(\frac{\epsilon}{1-\epsilon}\right)^2 = \delta^2. \quad (8)$$

The proof follows by equation (1), (7) and (8).

(2) ⇒ (3): We will verify the M-diskcyclic criterion with $D = D_1 = D_2 \subset M$ consisting of all sequences with finite support (sequences that only have a finite number of nonzero entries) It is clear that D is dense in M. By hypothesis, there exists a sequence of positive integers $\{n_p\} \to \infty$ such that $T^{n_p} M \subseteq M$ and $i, j \in N$ such that

$$\lim_{p\to\infty} \prod_{k=1}^{n_p} w_{m_j-k} = \infty \quad (9)$$

and

$$\lim_{p\to\infty} \frac{\prod_{k=0}^{n_p-1} w_{m_i+k}}{\prod_{k=1}^{n_p} w_{m_j-k}} = 0 \quad (10)$$

Let $y = \sum_{|m_j|\leq q} y_j e_{m_j} \in D$, then

$$\|B^{n_p} y\| \leq max\left\{\frac{1}{\prod_{k=1}^{n_p} w_{m_j-k}} : |m_j| \leq q\right\} \|y\|. \quad (11)$$

Where B is the backward shift operator. From equation (9), it is clear that $\lim_{p\to\infty} \|B^{n_p} y\| = 0$. Moreover, $T^{n_p} B^{n_p} y = y$. Now, let $x = \sum_{|m_i|\leq q} x_i e_{m_i} \in D$, then

$$\|T^{n_p} x\| \leq max\left\{\prod_{k=0}^{n_p-1} w_{m_i+k} : |m_i| \leq q\right\} \|x\|. \quad (12)$$

Then from equations (11) and (12), we get

$$\|T^{n_p} x\| \|B^{n_p} y\| \leq max\left\{\frac{\prod_{k=0}^{n_p-1} w_{m_i+k}}{\prod_{k=1}^{n_p} w_{m_j-k}} : |m_j|, |m_i| \leq q\right\} \|x\| \|y\|.$$

Since $\|x\| \|y\|$ is constant, then equation (10) implies that $\lim_{p\to\infty} \|T^{n_p} x\| \|B^{n_p} y\| = 0$. The proof follows by taking $x_p = B^{n_p} y$.

(3) ⇒ (1): Follows from Theorem 2.3.

The next theorem characterizes invertible bilateral forward weighted shifts that are subspace-disk transitive, first we need the following lemma.

Lemma 3.4. Let T be an invertible bilateral weighted shift, $\{n_k\}$ be a sequence of positive integers, $n_k \to \infty$ as $k \to \infty$ and M be a closed subspace such that $T^{n_k} M \subseteq M$. If there exist $i, j \in N$ such that $\|T^{n_k} e_{m_i}\| \|B^{n_k} e_{m_j}\| \to 0$ as $n_k \to \infty$, then $\|T^{n_k} e_{m_r}\| \|B^{n_k} e_{m_p}\| \to 0$ for all $r, p \in N$.

Proof: Since $T^{n_k} M \subseteq M$, the proof is similar to the proof of [15, Lemma 3.3.].

Theorem 3.5. Let T be an invertible bilateral forward weighted shift in the Hilbert space $l^2(Z)$ with a positive weight sequence $\{w_n\}_{n\in Z}$ and M be a subspace of $l^2(Z)$, then T is M-disk transitive if and only if there exists a sequence of positive integers $\{n_k\} \to \infty$ such that $T^{n_k} M \subseteq M$ and $i, j \in N$ such that

$$\lim_{k\to\infty} \prod_{s=1}^{n_k} \frac{1}{w_{m_i-s}} = 0$$

and

$$\lim_{k\to\infty} \left(\prod_{s=0}^{n_k-1} w_{m_j+s}\right)\left(\prod_{s=1}^{n_k} \frac{1}{w_{m_i-s}}\right) = 0.$$

Proof: To prove the "if" part , we will verify the M-diskcyclic criterion with $D = D1 = D2 \subset M$ consisting of all sequences with finite support, it is clear that D is dense in M. Let $x \in D$, then it suffices to show that $x = e_{m_r}$ and $y = e_{m_p}$ for some $r, p \in N$. Furthermore, by Lemma 2.4 and Lemma 3.4 it suffices to show that $\|B^{n_k} e_{m_i}\| \to 0$ and $\|T^{n_k} e_{m_j}\| \|B^{n_k} e_{m_i}\| \to 0$ for some $i, j \in N$. But that is clear because

$$\|B^{n_k} e_{m_i}\| = \prod_{s=1}^{n_k} \frac{1}{w_{m_i-s}} \to 0$$

and

$$\|T^{n_k} e_{m_j}\| \|B^{n_k} e_{m_i}\| = \left(\prod_{s=0}^{n_k-1} w_{m_j+s}\right)\left(\prod_{s=1}^{n_k} \frac{1}{w_{m_i-s}}\right) \to 0.$$

Moreover it is clear that $T^{n_k} B^{n_k} y = y$. By taking $x_k = B^{n_k} y$, it is clear that the conditions of M-diskcyclic criterion are satisfied.

The proof of the "only if" part follows from Theorem 3.3.

It is known that a weighted shift operator is invertible if and only if there exists $b > 0$ such that $|w_n| \geq b$ for all $n \in Z$.

The next theorem shows that the assumption of invertibility can be replaced by the assumption $w_n \geq b > 0$ for all $n < 0$.

Theorem 3.6. Let T be a bilateral forward weighted shift in the Hilbert space $l^2(Z)$ with a positive weight sequence $\{w_n\}_{n\in Z}$, $w_n \geq b > 0$ for all $n < 0$ and M be a subspace of $l^2(Z)$, then T is M-disk transitive if and only if there exists a sequence of positive integers $\{n_k\} \to \infty$ such that $T^{n_k} M \subseteq M$ and $i \in N$ such that $m_i \geq 0$ and

$$\lim_{k\to\infty} \prod_{s=1}^{n_k} \frac{1}{w_{m_i-s}} = 0$$

and

$$\lim_{k\to\infty} \left(\prod_{s=0}^{n_k-1} w_{m_i+s}\right)\left(\prod_{s=1}^{n_k} \frac{1}{w_{m_i-s}}\right) = 0.$$

Proof: For the "if" part, we will verify Theorem 3.3. Let $\epsilon > 0$, $q \in N$ and let $\delta_1, \delta_2 > 0$ (to be determined later), then there exist an arbitrary large n_k and $i \in N, m_i \geq 0$ such that

$$\prod_{s=1}^{n_k} \frac{1}{w_{m_i-s}} < \delta_1 \text{ and } \left(\prod_{s=0}^{n_k-1} w_{m_i+s}\right)\left(\prod_{s=1}^{n_k} \frac{1}{w_{m_i-s}}\right) < \delta_2 \qquad (13)$$

Let $n = n_k + m_i + q + 1$ (which ensure that $n + m_p \geq n_k + m_i + 1$ for all $|m_p| \leq q$).
By using Theorem 3.3, for all $m_p \in F$ with $|m_p| \leq q$ we have

$$\prod_{s=1}^{n} \frac{1}{w_{m_p-s}} = \prod_{s=1+m_p}^{n+m_p} \frac{1}{w_{-s}} = \prod_{s=1+m_p}^{m_i} \frac{1}{w_{-s}} + \prod_{s=1+m_i}^{n_k+m_i} \frac{1}{w_{-s}} + \prod_{s=n_k+m_i+1}^{n+m_p} \frac{1}{w_{-s}}$$

The first term of the above equation depends only on q, so we will let it a constant C_1, the second term is less than δ_1 by Equation (13) and the last term is less than $(1/b)^{2q}$. Therefore, we get

$$\prod_{s=1}^{n} \frac{1}{w_{m_p-s}} < C_1\delta_1(1/b)^{2q}$$

Since $C_1(1/b)^{2q}$ depends only on q, we will choose δ_1 is small enough such that $\delta_1 < (\epsilon_1 b^{2q})/C_1$ and we get

$$\prod_{s=1}^{n} \frac{1}{w_{m_p-s}} < \epsilon_1$$

Also, we have

$$\left(\prod_{s=0}^{n-1} w_{m_p+s}\right)\left(\prod_{s=1}^{n} \frac{1}{w_{m_p-s}}\right) = \left(\prod_{s=m_p}^{n+m_p-1} w_s\right)\left(\prod_{s=m_p+1}^{n+m_p} \frac{1}{w_{-s}}\right)$$
$$= \left(\prod_{s=m_p}^{m_i-1} w_s\right)\left(\prod_{s=m_i}^{n_k+m_i-1} w_s\right)\left(\prod_{s=n_k+m_i}^{n+m_p-1} w_s\right)\left(\prod_{s=m_p+1}^{m_i} \frac{1}{w_{-s}}\right) + \left(\prod_{s=m_i+1}^{n_k+m_i} \frac{1}{w_{-s}}\right) + \left(\prod_{s=n_k+m_i+1}^{n+m_p} \frac{1}{w_{-s}}\right).$$

By Equation (13), we get

$$\left(\prod_{s=0}^{n-1} w_{m_p+s}\right)\left(\prod_{s=1}^{n} \frac{1}{w_{m_p-s}}\right) < \left(\prod_{s=m_p}^{m_i-1} w_s\right)\left(\prod_{s=n_k+m_i}^{n+m_p-1} w_s\right)\left(\prod_{s=m_p+1}^{m_i} \frac{1}{w_{-s}}\right) + \left(\prod_{s=n_k+m_i+1}^{n+m_p} \frac{1}{w_{-s}}\right)\delta_2 < C_2\|T^{2q}\|C_1\left(\frac{1}{b}\right)^{2q}\delta_2$$

Since $C = C_2\|T^{2q}\|C_1(1/b)^{2q}$ is a constant depends only on q, then if we let δ_2 is small enough, such that $\delta_2 < \epsilon_2/C$ then for all $m_p \in F$ with $|m_p| \leq q$,

$$\left(\prod_{s=0}^{n-1} w_{m_p+s}\right)\left(\prod_{s=1}^{n} \frac{1}{w_{m_p-s}}\right) < \epsilon. \qquad (15)$$

The proof of the "if" part follows from Equation (14) and Equation (15). The "only if" part follows immediately from Theorem 3.3.

By the same way, we can characterize the subspace-disk transitive backward weighted shifts since they are unitarily equivalent to forward shifts.

Proposition 3.7. Let T be an invertible bilateral backward weighted shift in the Hilbert space $l^2(Z)$ with a positive weight sequence $\{w_n\}_{n\in Z}$ and M be a subspace of $l^2(Z)$, then T is M-disk transitive if and only if there exists a sequence of positive integers $\{n_k\} \to \infty$ such that $T^{n_k} M \subseteq M$ and $i, j \in N$ such that

$$\lim_{k\to\infty} \prod_{s=1}^{n_k} \frac{1}{w_{m_i+s}} = 0$$

and

$$\lim_{k\to\infty} \left(\prod_{s=0}^{n_k-1} w_{m_j-s} \right) \left(\prod_{s=1}^{n_k} \frac{1}{w_{m_i+s}} \right) = 0.$$

Proposition 3.8. Let T be a bilateral backward weighted shift in the Hilbert space $l^2(Z)$ with a positive weight sequence $\{w_n\}_{n\in Z}$, $w_n \geq b > 0$ for all $n < 0$ and M be a subspace of $l^2(Z)$, then T is M-disk transitive if and only if there exists a sequence of positive integers $\{n_k\} \to \infty$ such that $T^{n_k} M \subseteq M$ and $i \in N$ such that $m_i \geq 0$ and

$$\lim_{k\to\infty} \prod_{s=1}^{n_k} \frac{1}{w_{m_i+s}} = 0$$

and

$$\lim_{k\to\infty} \left(\prod_{s=0}^{n_k-1} w_{m_i-s} \right) \left(\prod_{s=1}^{n_k} \frac{1}{w_{m_i+s}} \right) = 0.$$

We have seen that bilateral weighted shifts are subspace-disk transitive if and only if some conditions are satisfied. Hence, one may ask the following question.

Question. Is there any bilateral weighted shift on $l^2(Z)$ that is subspace-diskcyclic but not subspace-disk transitive?

CONCLUSION

We have obtained that the bilateral weighted shift operator on $l^2(Z)$ is subspace-disk transitive for some subspaces of $l^2(Z)$ if and only if some certain conditions are satisfied.

REFERENCES

1. S. Rolewicz, On orbits of elements, Studia Math., 32 (1969), 17–22.
2. H. M. Hilden and L. J. Wallen, Some cyclic and non-cyclic vectors of certain operators, Indiana Univ. Math. J, 23 (1974), 557–565.
3. F. Bayart and E. Matheron, Dynamics of Linear Operators, Cambridge University Press 2009.
4. K. G. Grosse-Erdmann and A. Peris, Linear Chaos, in: Universitext, Springer, 2011.
5. Z. J. Zeana, Cyclic Phenomena of Operators on Hilbert Space; Thesis, University of Baghdad, 2002.
6. N. Bamerni and A. Kılıcman, M.S.M. Noorani, A review of some works in the theory of diskcyclic operators, Bull. Malays. Math. Sci. Soc., doi:10.1007/s40840-015-0137-x.
7. N. Bamerni, A. Kılıcman, Operators with Diskcyclic Vectors Subspaces, Journal of Taibah University for Science, 9 (2015), 414–419.
8. B.F. Madore and R. A. Martınez-Avendano, Subspace-hypercyclicity, J. Math. Anal. Appl., 373 (2011), 502–511.
9. R.R. Jimenez-Munguıa, R. A. Martınez-Avendano and A. Peris, Some questions about subspace-hypercyclic operators, J. Math. Anal. Appl.,408 (2013), 209-212.
10. C.M. Le, On subspace-hypercyclic operators, Proc. Amer. Math. Soc., B9(2011), 2847–2852.
11. H. Rezaei, Notes on subspace-hypercyclic operators, J. Math. Anal. Appl., 397 (2013), 428–433.
12. Z. Xian-Feng, S. Yong-Lu and Z. Yun-Hua, Subspace-supercyclicity and common subspace-supercyclic vectors, Journal of East China Normal University, 1 (2012), 106–112.
13. N. Bamerni, V. Kadets and A. Kılıcman, On subspace-diskcyclicity, http://arxiv.org/abs/1402.468.
14. H. N. Salas. Hypercyclic weighted shifts, Trans. Amer. Math. Soc., 347 (1995) 993–1004.
15. N. S. Feldman, Hypercyclicity and supercyclicity for invertible bilateral weighted shifts, Proc. Amer. Math. Soc., 131 (2003), 479–485.
16. N. Bamerni and A. Kılıcman, Subspace-hypercyclic weighted shifts, http://arxiv.org/abs/1501.02534.